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PHYSICS ACADEMY

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CSIR-NET-JRF

(MATHEMATICAL PHYSICS)

PREVIOUS YEAR'S QUESTIONS WITH ANSWER (CHAPTER-WISE)

- MATRIX ALGEBRA
- VECTOR ANALYSIS
- FOURIER SERIES, FOURIER & LAPLACE TRANSFROMATION
- COMPLEX ANALYSIS
- DIFFERENTIAL EQUATION
- PROBABILITY
- OTHER QUESTIONS

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MATRIX ALGEBRA

1. Which of the following matrices is an element of the group SU(2)? **[CSIR-JUNE-2011]**

(a) $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$
(b) $\begin{pmatrix} \frac{1+i}{\sqrt{3}} & \frac{-1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} & \frac{1-i}{\sqrt{3}} \end{pmatrix}$

(c) $\begin{pmatrix} 2+i & i \\ 3 & 1+i \end{pmatrix}$
(d) $\begin{pmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}$

2. Consider the matrix M = $\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$.

[CSIR-JUNE-2011]

A. The eigenvalues of M are

(a) 0, 1, 2 (b) 0, 0, 3 (c) 1, 1, 1 (d) – 1, 1, 3

B. The exponential of M simplifies to (I is the 3 × 3 identity matrix)

(a) $e^M = I + \left(\frac{e^3-1}{3}\right)M$
(b) $e^M = I + M + \frac{M^2}{2!}$

(c) $e^M = I + 3^3 M$
(d) $e^M = (e-1)M$

3. A 3 × 3 matrix M has Tr[M] = 6, Tr $[M^2]$=26 and Tr $[M^3]$=90. Which of the following can be a possible set of eigenvalues of M?

[CSIR- DEC-2011]

(a) {1,1, 4} (b) {1,0,7} (c) {-1,3, 4} (d) {2,2, 2}

4. The eigen values of the matrix A = $\begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{pmatrix}$ are. **[CSIR- JUNE-2012]**

(a) (1, 4,9) (b) (0, 7,7)

(c) (0,1,13) (d) (0,0,14)

5. The eigen values of the anti symmetric matrix, **[CSIR- JUNE-2012]**

$$A = \begin{pmatrix} 0 & -n_3 & n_2 \\ n_3 & 0 & -n_1 \\ -n_2 & n_1 & 0 \end{pmatrix}$$

When n_1 and n_2 and n_3 are the components of a unit vector, are.

(a) 0, i, -i (b) 0,1,-1 (c) 0,1+i,-1,-i (d) 0,0,0

6. Given a 2x2 unitary matrix U satisfying $U^\dagger U = UU^\dagger = 1$ with det U = $e^{i\varphi}$, one can construct a unitary matrix V($V^\dagger V = VV^\dagger = 1$)with det V =1 from it by.

[CSIR-DEC-2012]

(a) Multiplying U by $e^{-i\varphi/2}$
(b) Multiplying any single element of U by $e^{-i\varphi}$
(c) Multiplying any row or column of U by $e^{-i\varphi/2}$
(d) Multiplying U by $e^{-i\varphi}$

7. The approximation $\cos\theta \approx 1$ is valid up to 3 decimal places as long as $|\theta|$ is less than: (take$180^0/\pi \approx 57.29^0$). **[CSIR- JUNE-2013]**

(a) 1.28^0 (b) 1.81^0 (c) 3.28^0 (d) 4.01^0

8. Consider an $n \times n(n > 1)$ matrix A , in which A_{ij} is the product of the indices i and j (namely A_{ij}=ij). The matrix A. **[CSIR-DEC-2013]**

(a) Has one degenerate eigevalue with degeneracy (n-1).
(b) Has two degenerate eigenvalues with degeneracies 2 and (n-2)
(c) Has one degenerate eigenvalue with degeneracy *n*
(d) Does not have any degenerate eigenvalue

9. Consider the matrix. **[CSIR- JUNE-2014]**

$$M = \begin{pmatrix} 0 & 2i & 3i \\ -2i & 0 & 6i \\ -3i & -6i & 0 \end{pmatrix}$$

The eigenvalues of M are.

(a) -5, -2, 7 (b) -7, 0, 7 (c) -4*i*, 2*i*, 2*i* (d) 2, 3, 6

10. The column vector $\begin{pmatrix} a \\ b \\ a \end{pmatrix}$ is a simultaneous eigenvector of $A = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$ if. **[CSIR-DEC-2014]**

(a) b = 0 or a = 0 (b) b = a or b = -2a
(c) b = 2a or b = -a (d) b = a/2 or b = -a/2

11. The matrix $M = \begin{pmatrix} 1 & 3 & 2 \\ 3 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ satisfies the equation. **[CSIR-DEC-2016]**

(a) $M^3 - M^2 - 10M + 12I = 0$ (b) $M^3 + M^2 - 12M + 10I = 0$
(c) $M^3 - M^2 - 10M + 10I = 0$ (d) $M^3 + M^2 - 10M + 10I = 0$

12. Which of the following can not be the eigen values of a real 3x3 matrix: **[CSIR- JUNE-2017]**

(a) 2i,0,-2i (b) 1,1,1 (c) $e^{i\theta}, e^{-i\theta}, 1$ (d) i, 1, 0

13. Let $\sigma_x, \sigma_y, \sigma_z$ be the Pauli matrices and $x'\sigma_x + y'\sigma_y + z'\sigma_z = \exp\left(\frac{i\theta\sigma_z}{2}\right) \times [x\sigma_x + y\sigma_y + z\sigma_z] \exp\left(-\frac{i\theta\sigma_z}{2}\right)$, **[CSIR- DEC-2016]**

Then the coordinates are related as follows.

(a) $\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$

(b) $\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$

(c) $\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \cos\frac{\theta}{2} & \sin\frac{\theta}{2} & 0 \\ -\sin\frac{\theta}{2} & \cos\frac{\theta}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$

(d) $\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \cos\frac{\theta}{2} & -\sin\frac{\theta}{2} & 0 \\ \sin\frac{\theta}{2} & \cos\frac{\theta}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$

14. Let A be a non-singular 3x3 matrix, the columns of which are denoted by the vectors $\vec{a}, \vec{b}$ and $\vec{c}$, respectively. Similarly, $\vec{u}, \vec{v}$ and $\vec{w}$ denote the vectors that form the corresponding columns of $(A^T)^{-1}$. Which of the following is true?

[CSIR-DEC-2016]

(a) $\vec{u}.\vec{a} = 0, \vec{u}.\vec{b} = 0, \vec{u}.\vec{c} = 1$

(b) $\vec{u}.\vec{a} = 0, \vec{u}.\vec{b} = 1, \vec{u}.\vec{c} = 0$

(c) $\vec{u}.\vec{a} = 1, \vec{u}.\vec{b} = 0, \vec{u}.\vec{c} = 0$

(d) $\vec{u}.\vec{a} = 0, \vec{u}.\vec{b} = 0, \vec{u}.\vec{c} = 0$

15. Consider the matrix equation **[CSIR-DEC-2017]**

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 2 & b & 2c \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

The condition for existence of a non-trivial solution and the corresponding normalized solution (upto a sign) is

(a) $b = 2c$ and (x,y,z) = $\frac{1}{\sqrt{6}}(1, -2, 1)$

(b) $c = 2b$ and (x,y,z) = $\frac{1}{\sqrt{6}}(1, 1, -2)$

(c) $c = b + 1$ and (x,y,z) = $\frac{1}{\sqrt{6}}(2, -1, -1)$

(d) $b = c + 1$ and (x,y,z) = $\frac{1}{\sqrt{6}}(1, -2, 1)$

16. Which of the following statements is true for a 3x3 real orthogonal matrix with determinant +1? **[CSIR- JUNE-2017]**

(a) The modulus of each of its eigenvalues need not be 1, but their product must be 1

(b) At least one of its eigenvalues is +1

(c) All of its eigenvalues must be real

(d) None of its eigenvalues must be real

17. One of the eigenvalues of the matrix e^A is e^a, where A = $\begin{pmatrix} a & 0 & 0 \\ 0 & 0 & a \\ 0 & a & 0 \end{pmatrix}$. The product of the other two eigenvalues of e^A is. **[CSIR- DEC-2018]**

(a) e^{2a} (b) e^{-a} (c) e^{-2a} (d) 1

18. A 4×4 complex matrix A satisfies the relation $A^{\dagger}A = 4I$, where I is the 4×4 identity matrix. The number of independent real parameters of A is.

[CSIR-DEC-2018]

(a) 32 (b) 10 (c) 12 (d) 16

19. The element of a 3 × 3 matrix Aare the product if its row and column indices $A_{ij} = ij$ (where i, j= 1,2,3,…..). The eigenvalues of A are. **[CSIR-JUNE-2019]**

(a) (7,7,0) (b) (7,4,3) (c) (14,0,0) (d) $(\frac{14}{3}, \frac{14}{3}, \frac{14}{3})$

20. If the rank of n × n matrix A is m , where m and n are positive integers with 1 ≤ m ≤ n , then the rank of matrix A is. **[CSIR-DEC-2019]**

(a) m (b) m-1 (c) 2m (d) m-2

21. The eigenvalues of the 3×3 matrix $M = \begin{pmatrix} a^2 & ab & ac \\ ab & b^2 & bc \\ ac & bc & c^2 \end{pmatrix}$ are. **[CSIR-NOV-2020]**

(a) $a^2 + b^2 + c^2$,0 , 0 (b) $b^2+c^2, a^2, 0$

(c) $a^2 + b^2, c^2$,0 (d) a^2+c^2 ,b^2,0

VECTOR ANALYSIS

1. Let $\vec{a}$ and $\vec{b}$ be two distinct three dimensional vectors. Then the component of $\vec{b}$ that is perpendicular to $\vec{a}$ is given by. **[CSIR-JUNE-2011]**

(a) $\frac{\vec{a}\times(\vec{b}\times\vec{a})}{a^2}$ (b) $\frac{\vec{b}\times(\vec{a}\times\vec{b})}{b^2}$

(c) $\frac{(\vec{a}.\vec{b})b}{b^2}$ (d) $\frac{(\vec{b}.\vec{a})\vec{a}}{a^2}$

2. The equation of the plane that is tangent to the surface xyz = 8 at the point (1, 2, 4)is **[CSIR- DEC-2011]**

(a) x +2y + 4z=12 (b) 4x +2y +z =12

(c) x +4y +2 =0 (d) x + y +z =7

3. A vector perpendicular to any vector that lies on the plane defined by x + y + z = 5 , is. **[CSIR-JUNE-2012]**

(a) $\hat{i} + \hat{j}$ (b) $\hat{j} + \hat{k}$ (c) $\hat{i} + \hat{j} + \hat{k}$ (d) $2\hat{i} + 3\hat{j} + 5\hat{k}$

4. Which of the following limits exists? **[CSIR-JUNE-2012]**

(a) $\lim_{N\to\infty}\left(\sum_{m=1}^{N}\frac{1}{m} + \ln N\right)$ (b) $\lim_{N\to\infty}\left(\sum_{m=1}^{N}\frac{1}{m} - \ln N\right)$

(c) (a) $\lim_{N\to\infty}\left(\sum_{m=1}^{N}\frac{1}{\sqrt{m}} - \ln N\right)$ (d) $\lim_{N\to\infty}\sum_{m=1}^{N}\frac{1}{m}$

5. The unit normal vector of the point $\left[\frac{a}{\sqrt{3}},\frac{b}{\sqrt{3}},\frac{c}{\sqrt{3}}\right]$on the surface of the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2} = 1$ is. **[CSIR-DEC-2012]**

(a) $\frac{bc\hat{i}+ca\hat{j}+ab\hat{k}}{\sqrt{a^2+b^2+c^2}}$ (b) $\frac{a\hat{i}+b\hat{j}+c\hat{k}}{\sqrt{a^2+b^2+c^2}}$

(c) $\frac{b\hat{i}+c\hat{j}+a\hat{k}}{\sqrt{a^2+b^2+c^2}}$ (d) $\frac{\hat{i}+\hat{j}+\hat{k}}{\sqrt{3}}$

6. A unit vector $\hat{n}$ on the xy -plane is at an angle of 120^0 with respect to $\hat{\imath}$. The angle between the vectors $\vec{u} = a\hat{\imath} + b\hat{n}$ and $\vec{v} = a\hat{n} + b\hat{\imath}$ will be 60^0 if. **[CSIR- JUNE-2013]**

(a) $b = \sqrt{3}a / 2$ (b) $b = 2a / \sqrt{3}$

(c) $b = a / 2$ (d) $b = a$

7. If $\vec{A} = \hat{\imath}yz + \hat{\jmath}xz + \hat{k}xy$, then the integral $\oint_C \vec{A}.d\vec{l}$ (where C is along the perimeter of a rectangular area bounded by x = 0, $x = a$ and y = 0, $y = b$) is. **[CSIR-DEC-2013]**

(a) $\frac{1}{2}(a^3 + b^3)$ (b) $\pi(ab^2 + a^2b)$

(c) $\pi(a^3 + b^3)$ (d) 0

8. Let $\vec{r}$ denote the position vector of any point in three-dimensional space, and r = | $\vec{r}$ | Then. **[CSIR-DEC-2014]**

(a) $\vec{\nabla}.\vec{r} = 0$ and $\vec{\nabla} \times \vec{r} = \vec{r}/r$ (b) $\vec{\nabla}.\vec{r} = 0$ and $\nabla^2 r = 0$

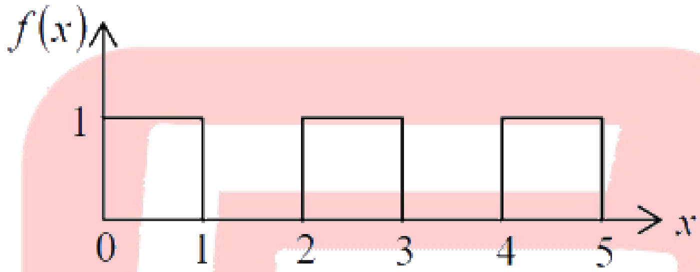
(c) $\vec{\nabla}.\vec{r} = 0$ and $\nabla^2 \times \vec{r} = \vec{r}/r^2$ (d) $\vec{\nabla}.\vec{r} = 3$ and $\vec{\nabla} \times \vec{r} = 0$

9. Consider the three vectors $\vec{v}_1 = 2\hat{\imath} + 3\hat{k}, \vec{v}_2 = \hat{\imath} + 2\hat{\jmath} + 2\hat{k}$ and $\vec{v}_3 = 5\hat{\imath} + 2\hat{\jmath} + a\hat{k}$ where $\hat{\imath}, \hat{\jmath}$ and $\hat{k}$ are the standard unit vectors in a three-dimensional Euclidean space. These vectors will be linearly dependent if the value of a is. **[CSIR- JUNE-2018]**

(a) 31/4 (b) 23/4 (c) 27/4 (d) 0

FOURIER SERIES, FOURIER & LAPLACE TRANSFROMATION

1. The graph of the function $f(x) = \begin{cases} 1 & \text{for } 2n \le x \le 2n+1 \\ 0 & \text{for } 2n+1 \le x \le 2n+2 \end{cases}$ where n = (0, 1, 2,……….) is shown below, Its Laplace $\tilde{f}(s)$ is. **[CSIR-DEC-2011]**



(a) $\frac{1+e^{-s}}{s}$ (b) $\frac{1-e^{-s}}{s}$ (a) $\frac{1}{s(1+e^{-s})}$ (d) $\frac{1}{s(1-e^{-s})}$

2. Consider a sinusoidal waveform of amplitude $1V$ and frequency f_0 Starting from an arbitrary initial time, the waveform is sampled at intervals of $\frac{1}{2f_0}$. If the corresponding Fourier spectrum peaks at a frequency $\bar{f}$ and an amplitude $\bar{A}$, them. **[CSIR-JUNE-2012]**

(a) $\bar{f} = 2f_0$ and $\bar{A} = 1V$ (b) $\bar{f} = 2f_0 \le \bar{A} \le 1V$

(c) $\bar{f} = 0$ and $\bar{A} = 1V$ (d) $\bar{f} = \frac{f_0}{2}$ and $\bar{A} = \frac{1}{\sqrt{2}}V$

3. The inverse Laplace transforms of $\frac{1}{s^2(s+1)}$ is. **[CSIR-JUNE-2013]**

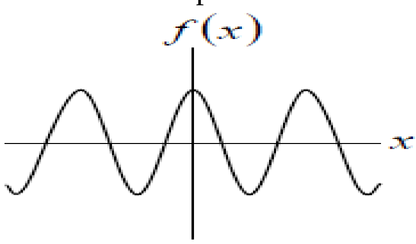
(a) $\frac{1}{2}t^2e^{-t}$ (b) $\frac{1}{2}t^2 + 1 - e^{-t}$

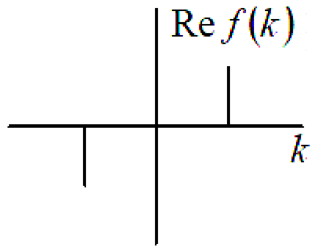
(c) $t - 1 + e^{-t}$ (d) $\frac{1}{2}t^2(1 - e^{-t})$

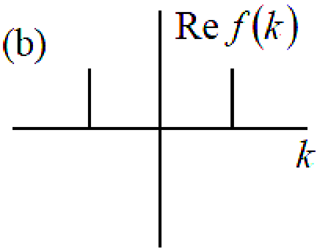
4. The Fourier transform of the derivative of the Dirac δ - function, namely $\delta'(x)$, is proportional to. **[CSIR-DEC-2013]**

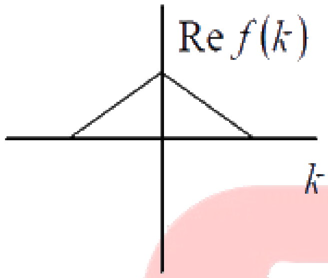
(a) 0 (b) 1 (c) sin k (d) ik

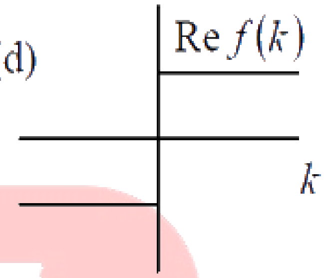
5. The graph of a real periodic function $f(x)$for the range $[-\infty, \infty]$is shown in the figure. Which of the following graphs represents the real part of its Fourier transform? **[CSIR-JUNE-2014]**



(a)


(b)


(c)


(d)


6. The Laplace transform of $6t^3 + 3\sin 4t$ is. **[CSIR-JUNE-2015]**

(a) $\frac{36}{s^4} + \frac{12}{s^2+16}$
(b) $\frac{36}{s^4} + \frac{12}{s^2-16}$
(c) $\frac{18}{s^4} + \frac{12}{s^2-16}$
(d) (a) $\frac{36}{s^3} + \frac{12}{s^2+16}$

7. The Fourier transform of f(x) is $\tilde{f}(k) = \int_{-\infty}^{+\infty} dx e^{ikx} f(x)$.
If f(x) = $\alpha\delta(x) + \beta\delta'(x) + \gamma\delta''(x)$, where $\delta(x)$ is the Dirac delta-function (and prime denotes derivative), what is $\tilde{f}(k)$? **[CSIR-DEC-2015]**

(a) $\alpha + i\beta k + i\gamma k^2$
(b) $\alpha + \beta k - \gamma k^2$
(c) $\alpha - i\beta k - \gamma k^2$
(d) $i\alpha + \beta k - i\gamma k^2$

8. What is the Fourier transform $\int dx e^{ikx} f(x)$ of. **[CSIR-JUNE-2016]**

$$f(x) = \delta(x) + \sum_{n=1}^{\infty} \frac{d^n}{dx^n}\delta(x)$$

What $\delta(x)$ is the Dirac delta-function?

(a) $\frac{1}{1-ik}$
(b) $\frac{1}{1+ik}$
(c) $\frac{1}{k+i}$
(d) $\frac{1}{k-i}$

9. The Laplace transform of $f(t) = \begin{cases} \frac{t}{T}, & 0 < t < T \\ 1 & t > T \end{cases}$ is. **[CSIR-DEC-2016]**

(a) $\frac{-(1-e^{-sT})}{S^2 T}$
(b) $\frac{(1-e^{-sT})}{S^2 T}$
(c) $\frac{(1+e^{-sT})}{S^2 T}$
(d) $\frac{(1-e^{sT})}{S^2 T}$

10. The Fourier transform $\int_{-\infty}^{\infty} dx f(x) e^{ikx}$ of the function $f(x) = \frac{1}{x^2+2}$ is.
[CSIR-DEC-2016]

(a) $\sqrt{2}\pi e^{-\sqrt{2}|k|}$
(b) $\sqrt{2}\pi e^{-\sqrt{2}k}$
(c) (a) $\frac{\pi}{\sqrt{2}} e^{-\sqrt{2}k}$
(d) (a) $\frac{\pi}{\sqrt{2}} e^{-\sqrt{2}|k|}$

11. Consider the differential equation $\frac{dy}{dt} + ay = e^{-bt}$ with the initial condition y(0) = 0. Then the Laplace transform Y(s) of the solution y(t) is. **[CSIR-DEC-2016]**

(a) $\frac{1}{(s+a)(s+b)}$ (b) $\frac{1}{b(s+a)}$ (c) $\frac{1}{a(s+b)}$ (d) $\frac{e^{-a}-e^{-b}}{b-a}$

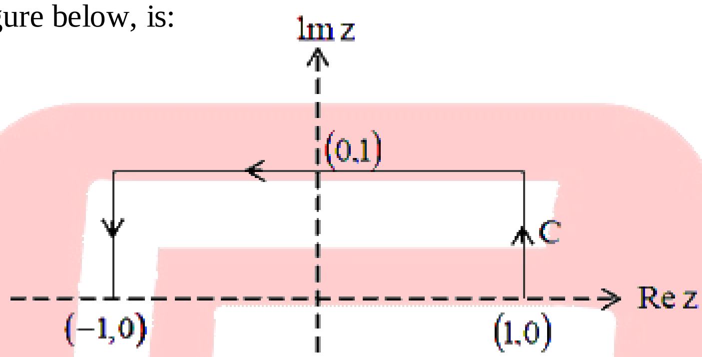
12. The Fourier transform $\int_{-\infty}^{\infty} dx f(x) e^{ikx}$ of the function $f(x) = e^{-|x|}$.

[CSIR-JUNE-2018]

(a) $-\frac{2}{1+k^2}$ (b) $-\frac{1}{2(1+k^2)}$ (c) $\frac{2}{1+k^2}$ (d) $\frac{2}{(2+k^2)}$

COMPLEX ANALYSIS

1. The value of the integral $\int_C dz\, z^2 e^z$, where C is an open contour in the complex z -plane as shown in the figure below, is: **[CSIR-JUNE-2011]**



(a) $\frac{5}{e} + e$ (b) e- $\frac{5}{e}$ (c) $\frac{5}{e} - e$ (d) - $\frac{5}{e} - e$

2. Which of the following is an analytic function of the complex variable z =x+iy in the domain |z|< 2? **[CSIR-JUNE-2011]**

(a) $(3+x-iy)^7$ (b) $(1+x+iy)^4 (7-x-iy)^3$

(c) $(1-x-iy)^4 (7-x+iy)^3$ (d) $(x+iy-1)^{1/2}$

3. The first few terms in the Laurent series for $\frac{1}{(z-1)(z-2)}$ in the region $1 \le |z| \le 2$ and around z = 1 is. **[CSIR-JUNE-2012]**

(a) $\frac{1}{2}[1 + z + z^2 + \cdots]\left[1 + \frac{z}{2} + \frac{z^2}{4} + \frac{z^3}{8} + \cdots\right]$

(b) $\frac{1}{1-z} - z - (1-z)^2 + (1-z)^3 + \cdots$

(c) $\frac{1}{z^2}\left[1 + \frac{1}{z} + \frac{1}{z^2} + \cdots\right]\left[1 + \frac{2}{z} + \frac{4}{z^2} + \cdots\right]$

(d) $2(z-1) + 5(z-1)^2 + 7(z-1)^3 + \cdots$

4. Let u(x,y) = $x + \frac{1}{2}(x^2 - y^2)$be the real part of analytic function f(z) of the complex variable z = x+iy The imaginary part of f (z) is: **[CSIR-JUNE-2012]**

(a) y +xy (b) xy (c) y (d) y^2-x^2

5. The Taylor expansion of the function ln(cosh x)where x is real, about the point $x = 0$, starts with the following terms: **[CSIR -DEC-2012]**

(a) $-\frac{1}{2}x^2 + \frac{1}{12}x^4 + \cdots$ (b) $\frac{1}{2}x^2 - \frac{1}{12}x^4 + \cdots$

(c) $-\frac{1}{2}x^2 + \frac{1}{6}x^4 + \cdots$ (d) $\frac{1}{2}x^2 + \frac{1}{6}x^4 + \cdots$

6. The value of the integral $\int_C \frac{z^3 dz}{(z^2-5z+6)}$, where C is a closed contour defined by the equation $2|z| - 5 = 0$, traversed in the anti-clockwise direction, is **[CSIR -DEC-2012]**

(a) $-16\pi i$ (b) $16\pi i$ (c) $8\pi i$ (d) $2\pi i$

7. With $z = x + iy$, which of the following functions $f(x,y)$ is NOT a (complex) analytic function of z ? **[CSIR-JUNE-2013]**

(a) $f(x,y) = (x+iy-8)^3(4+x^2-y^2+2ixy)^7$

(b) $f(x,y) = (x+iy)^7(1-x-iy)^3$

(c) $f(x,y) = (x^2-y^2+2ixy)^5$

(d) $f(x,y) = (1-x+iy)^4(2+x+iy)^6$

8. Which of the following functions cannot be the real part of a complex analytic function of $z = x+iy$? **[CSIR-DEC-2013]**

(a) $x^2 y$ (b) $x^2 - y^2$ (c) $x^3 - 3xy^2$ (d) $3x^2y - y - y^3$

9. Given that the integral $\int_0^\infty \frac{dx}{y^2+x^2} = \frac{\pi}{2y}$, the value of $\int_0^\infty \frac{dx}{(y^2+x^2)^2}$ is. **[CSIR-DEC-2013]**

(a) $\frac{\pi}{y^3}$ (b) $\frac{\pi}{4y^3}$ (c) $\frac{\pi}{8y^3}$ (d) $\frac{\pi}{2y^3}$

10. If C is the contour defined by $|z| = \frac{1}{2}$, the value of the integral $\int_C \frac{dz}{\sin^2 z}$ is. **[CSIR -JUNE-2014]**

(a) ∞ (b) $2\pi i$ (c) 0 (d) πi

11. The principal value of the integral $\int_{-\infty}^{\infty} \frac{\sin(2x)}{x^3} dx$ is. **[CSIR-DEC-2014]**

(a) -2π (b) $-\pi$ (c) π (d) 2π

12. The Laurent series expansion of the function $f(z) = e^z + e^{1/z}$ about $z = 0$ is given by. **[CSIR-DEC-2014]**

(a) $\sum_{n=-\infty}^{\infty} \frac{z^n}{n!}$ for all $|z| < \infty$

(b) $\sum_{n=0}^{\infty} \left(z^n + \frac{1}{z^n}\right)\frac{1}{n!}$ only if $0 < |z| < 1$

(c) $\sum_{n=0}^{\infty} \left(z^n + \frac{1}{z^n}\right)\frac{1}{n!}$ only if $0 < |z| < \infty$

(d) $\sum_{n=-\infty}^{\infty} \frac{z^n}{n!}$ only if $0 < |z| < 1$

13. Consider the function $f(z) = \frac{1}{z}\ln(1-z)$ of a complex variable $z = re^{i\theta}$ $(r \geq 0, -\infty < \theta < \infty)$. The singularities of f(z) are as follows: **[CSIR-DEC-2014]**

(a) Branch points at $z = 1$ and $z = \infty$; and a pole at $z = 0$ only for $0 \leq \theta < 2\pi$

(b) Branch points at $z = 1$ and $z = \infty$; and a pole at $z = 0$ for all θ other than $0 \leq \theta < 2\pi$

(c) Branch points at $z = 1$ and $z = \infty$; and a pole at $z = 0$ for all θ

(d) Branch points at $z = 1$, $z = 1$ and $z = \infty$

14. The value of integral $\int_{-\infty}^{\infty} \frac{dx}{1+x^4}$. **[CSIR-JUNE-2015]**

(a) $\frac{\pi}{\sqrt{2}}$ (b) $\frac{\pi}{2}$ (c) $\sqrt{2}\pi$ (d) 2π

15. The radius of convergence of the Taylor series expansion of the function $\frac{1}{\cosh(x)}$ around $x = 0$, is. **[CSIR-JUNE-2016]**

(a) ∞ (b) π (c) $\frac{\pi}{2}$ (d) 1

16. The value of the contour integral $\frac{1}{2\pi i}\oint_C \frac{e^{4z}-1}{\cosh(z)-2\sin h(z)}dz$ around the unit circle Ctraversed in the anti-clockwise direction, is **[CSIR-JUNE-2016]**

(a) 0 (b)2 (c) $\frac{-8}{\sqrt{3}}$ (d) -tanh$\left(\frac{1}{2}\right)$

17. The Gauss hypergeometric function F(a,b,c,z), defined by the Taylor series expansion around z = 0 as F(a,b,c,z) = $\sum_{n=0}^{\infty}\frac{a(a+1)\ldots(a+n-1)b(b+1)\ldots(b+n-1)}{c(c+1)\ldots(c+n-1)n!}z^n$, satisfies the recursion relation. **[CSIR-JUNE-2016]**

(a) $\frac{d}{dz}F(a,b,c;z) = \frac{c}{ab}F(a-1,b-1,c-1;z)$

(b) $\frac{d}{dz}F(a,b,c;z) = \frac{c}{ab}F(a+1,b+1,c+1;z)$

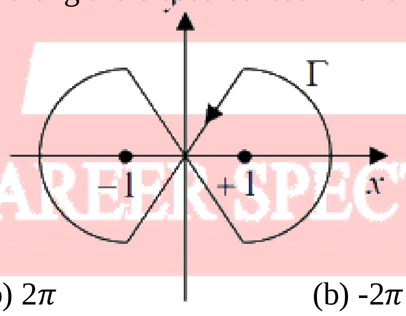
(c) $\frac{d}{dz}F(a,b,c;z) = \frac{ab}{c}F(a-1,b-1,c-1;z)$

(d) $\frac{d}{dz}F(a,b,c;z) = \frac{ab}{c}F(a+1,b+1,c+1;z)$

18. Let $u(x,y) = e^{ax}\cos(by)$ be the real part of a function $f(z) = u(x,y) + iv(x,y)$ of the complex variable $z = x + iy$, where a,b are real constants and $a \neq 0$. The function f(z) is complex analytic everywhere in the complex plane if and only if. **[CSIR-DEC-2016]**

(a) b = 0 (b) $b = \pm a$ (c) $b = \pm 2\pi a$ (d) $b = a \pm 2\pi$

19. The integral $\oint_\Gamma \frac{ze^{i\pi z/2}}{z^2-1}dz$ along the closed contour Γ shown in the figure is. **[CSIR- DEC-2016]**



(a) 0 (b) 2π (b) -2π (d) $4\pi i$

20. Consider the real function $f(x) = 1/(x^2+4)$. The Taylor expansion of f(x) about x = 0 converges. **[CSIR-DEC-2016]**

(a) For all values of x

(b) For all values of x except $x = \pm 2$

(c) In the region -2 < x < 2

(d) For x >2 and x < -2

21. What is the value of a for which $f(x,y) = 2x + 3(x^2 - y^2) + 2i(3xy + ay)$ is an analytic function of complex variable z = x+ iy. **[CSIR-JUNE-2018]**

(a) 1 (b) 0 (c) 3 (d) 2

22. In the function $P_x(x)e^{-x^2}$ of a real variable x, $P_n(x)$ is polynomial of degree n . The maximum number of extrema that this function can have is. **[CSIR-JUNE-2018]**

(a) n+ 2 (b) n-1 (c) n+1 (d) n

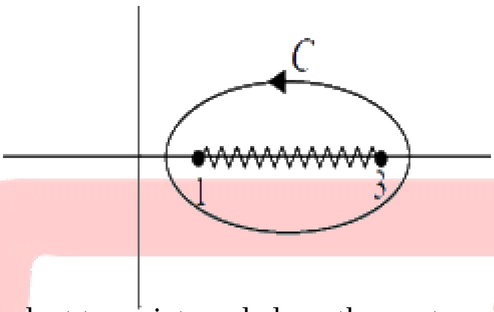
23. The value of the integral $\oint_C \frac{dz}{z}\frac{\tan h\, 2z}{\sin \pi z}$, where C is a circle of radius $\frac{\pi}{2}$, counter-clockwise, with centre at $z = 0$, is **[CSIR-JUNE-2018]**

(a) 4 (b) $4i$ (c) $2i$ (d) 0

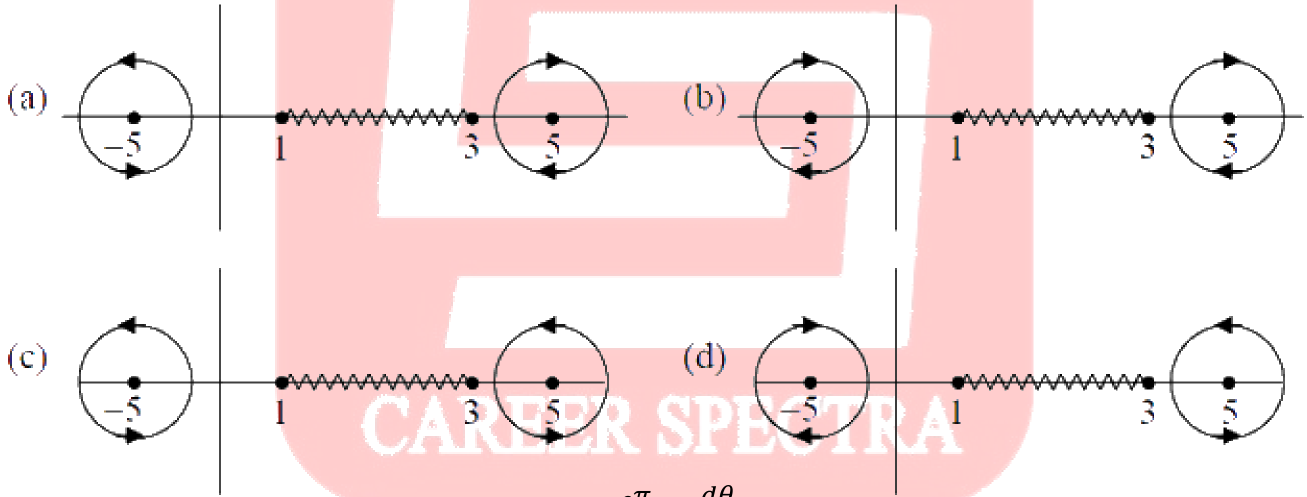
24. The contour C of the following integral **[CSIR-DEC-2018]**

$$\oint_z dx \frac{\sqrt{(z-1)(z-3)}}{(z^2-25)^3}$$

in the complex z plane is shown in the figure below.



This integral is equivalent to an integral along the contours



25. The value of the definite integral $\int_0^{\pi} \frac{d\theta}{5+4\cos\theta}$ is. **[CSIR JUNE-2019]**

(a) $\frac{4\pi}{3}$ (b) $\frac{2\pi}{3}$ (c) π (d) $\frac{\pi}{3}$

26. Let C be the circle of radius $\pi/4$, centered at z = $\frac{1}{4}$ in the complex z-plane that is traversed counter-clockwise. The value of the contour integral $\oint_C \frac{z^2}{sin^2 4z} dz$ is. **[CSIR DEC-2019]**

(a) 0 (b) $\frac{i\pi^2}{4}$ (c) $\frac{i\pi^2}{16}$ (d) $\frac{i\pi}{4}$

27. A function of a complex variable 'z' is defined by the integral f(z) = $\oint_\Gamma \frac{\omega^2-2}{\omega-z} d\omega$. Where Γ is a circular contour of radius 3, centred at origin running counter-clockwise in the w-plane.

The value of the function at z = (2 - i) is.

[CSIR-NOV-2020]

(a) 0 (b) 1 – 4i (c) $8\pi + 2\pi i$ (d) $-\frac{2}{\pi} - \frac{i}{2\pi}$

DIFFERENTIAL EQUATION

1. Let $p_n(x)$ (where n=0,1,2,......) be a polynomial of degree n with real coefficients, defined in the interval $2 \le n \le 4$. If $\int_2^4 p_n(x)p_m(x)dx = \delta_{nm}$, then . **[CSIR-JUNE-2011]**

(a) $p_0(x) = \frac{1}{\sqrt{2}}$ and $p_1(x) = \sqrt{\frac{3}{2}}(-3 - x)$

(b) $p_0(x) = \frac{1}{\sqrt{2}}$ and $p_1(x) = \sqrt{3}(3 + x)$

(c) $p_0(x) = \frac{1}{2}$ and $p_1(x) = \sqrt{\frac{3}{2}}(3 - x)$

(d) $p_0(x) = \frac{1}{\sqrt{2}}$ and $p_1(x) = \sqrt{\frac{3}{2}}(3 - x)$

2. The generating function $F(x,t) = \sum_{n=0}^{\infty} P_n(x)t^n$ for the Legendre polynomials $P_n(x)$ is $F(x,t) = (1 - 2xt + t^2)^{-1/2}$. The value of $P_3(-1)$ is. **[CSIR-DEC-2011]**

(a) 5 / 2 (b) 3/ 2 (c) +1 (d) -1

3. Let $x_1(t)$ and $x_2(t)$ be two linearly independent solutions of the differential equation $\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + f(t)x = 0$ and let $w(t) = x_1(t)\frac{dx_2(t)}{dt} - x_2(t)\frac{dx_1(t)}{dt}$. If w(0) = 1, then w(1) is given by,

[CSIR-DEC-2011]

(a) 1 (b) e^2 (c) 1/ e (d) $1/e^2$

4. Let $y(x)$ be a continuous real function in the range 0 and 2π, satisfying the inhomogeneous differential equation: $\sin x\frac{d^2y}{dx^2} + \cos x\frac{dy}{dx} = \delta\left(x - \frac{\pi}{2}\right)$.

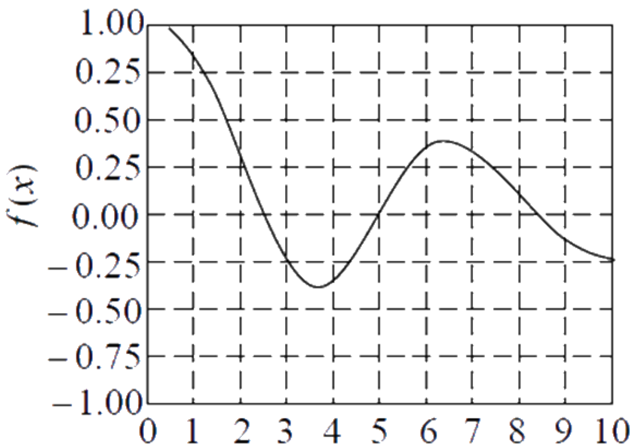
The value of *dyldx* at the point $x = \pi/2$. **[CSIR-JUNE-2012]**

(a) is continuous (b) has a discontinuity of 3

(c) has a discontinuity of 1/3 (d) has a discontinuity of 1

5. The graph of the function $f(x)$ shown below is best described by:

[CSIR-DEC-2012]



(a) The Bessel function $J_0(x)$ (b) cos x
(c) e^{-x} cos x (d) 1/x cos x

6. Given that $\sum_{n=0}^{\infty} H_n(x)\frac{t^n}{n!} = e^{-t^2+2tx}$ the value of $H_4(0)$ is. **[CSIR-JUNE-2013]**
(a) 12 (b) 6 (c) 24 (d) – 6

7. The solution of the partial differential equation **[CSIR-JUNE-2013]**

$$\frac{\partial^2}{\partial t^2}u(x,t) - \frac{\partial^2}{\partial x^2}u(x,t) = 0$$

satisfying the boundary conditions u(0,t) = 0 = u(L,t) and initial conditions $u(x,0) = \sin(\pi x/L)$ and $\frac{\partial}{\partial t}u(x,t)|_{t=0} = \sin\left(\frac{2\pi x}{L}\right)$ is.
(a) $\sin\left(\frac{\pi x}{L}\right)\cos\left(\frac{\pi x}{L}\right) + \frac{L}{2\pi}\sin\left(\frac{2\pi x}{L}\right)\cos\left(\frac{2\pi t}{L}\right)$
(b) $2\sin\left(\frac{\pi x}{L}\right)\cos\left(\frac{\pi t}{L}\right) - \sin\left(\frac{\pi x}{L}\right)\cos\left(\frac{2\pi t}{L}\right)$
(c) $\sin\left(\frac{\pi x}{L}\right)\cos\left(\frac{2\pi t}{L}\right) + \frac{L}{\pi}\sin\left(\frac{2\pi x}{L}\right)\cos\left(\frac{\pi t}{L}\right)$
(d) $\sin\left(\frac{\pi x}{L}\right)\cos\left(\frac{\pi t}{L}\right) + \frac{L}{2\pi}\sin\left(\frac{2\pi x}{L}\right)\cos\left(\frac{2\pi t}{L}\right)$

8. The solution of the differential equation. **[CSIR-JUNE-2013]**

$$\frac{dx}{dt} = x^2$$

with the initial condition *x*(0) = 1 will blow up as *t* tends to
(a) 1 (b) 2 (c) 1/2 (d) ∞

9. Consider the differential equation **[CSIR-JUNE-2014]**

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + x = 0$$

with the initial conditions *x*(0) = 0 and $\dot{x}(0) = 1$. The solution *x*(t) attains its maximum value when *t* is
(a) 1/2 (b) 1 (c) 2 (d) ∞

10. Given $\sum_{n=0}^{\infty} P_n(x)t^n = (1-2xt+t^2)^{-1/2}$, for | t | < 1, the value of P_5 (-1) is.
[CSIR-JUNE-2014]
(a) 0.26 (b) 1 (c) 0.5 (d) -1

11. The function $f(x) = \sum_{n=0}^{\infty}\frac{(-1)^n}{n!(n+1)!}\left(\frac{x}{2}\right)^{2n+1}$, satisfies the differential equation.
[CSIR-DEC-2014]
(a) $x^2\frac{d^2f}{dx^2} + x\frac{df}{dx} + (x^2+1)f = 0$
(b) $x^2\frac{d^2f}{dx^2} + 2x\frac{df}{dx} + (x^2-1)f = 0$
(c) $x^2\frac{d^2f}{dx^2} + x\frac{df}{dx} + (x^2-1)f = 0$
(d) $x^2\frac{d^2f}{dx^2} - x\frac{df}{dx} + (x^2-1)f = 0$

12. Consider the differential equation $\frac{d^2x}{dt^2} - 3\frac{dx}{dt} + 2x = 0$. If *x*= 0 at *t* = 0 and *x*= 1 at *t*=1, the value of *x* at *t* = 2 is: **[CSIR-JUNE-2015]**

(a) $e^2 + 1$ (b) $e^2 + e$ (c) e + 2 (d) 2e

13. Let f(x,t) be a solution of the wave equation $\frac{\partial^2 f}{\partial t^2} = v^2 \frac{\partial^2 f}{\partial x^2}$ in 1 -dimension. If at t = 0, f (x, 0) $= e^{-x^2}$ and $\frac{\partial f}{\partial t}$ (x, 0) = 0 for all x , then *f(x,t)* for all future times t > 0 is described by.

[CSIR-JUNE-2015]

(a) $e^{-(x^2-v^2t^2)}$ (b) $e^{-(x-vt)^2}$

(c) $\frac{1}{4}e^{-(x-vt)^2} + \frac{3}{4}e^{-(x+vt)^2}$ (d) $\frac{1}{4}\left(e^{-(x-vt)^2} + e^{-(x+vt)^2}\right)$

14. The solution of the differential equation $\frac{dx}{dt} = 2\sqrt{1-x^2}$, with initial condition $x = 0$ at $t = 0$ is.

[CSIR-DEC-2015]

(a) $x = \begin{cases} \sin 2t, & 0 \le t \le \frac{\pi}{4} \\ \sinh 2t, & t \ge \frac{\pi}{4} \end{cases}$ (b) $x = \begin{cases} \sin 2t, & 0 \le t < \frac{\pi}{2} \\ 1, & t \ge \frac{\pi}{4} \end{cases}$

(c) $x = \begin{cases} \sin 2t, & 0 \le t < \frac{\pi}{4} \\ 1, & t \ge \frac{\pi}{4} \end{cases}$ (d) $x = 1 - \cos 2t, t \ge 0$

15. The Hermite polynomial $H_n(x)$ satisfies the differential equation.

$$\frac{d^2H_n}{dx^2} - 2x\frac{dH_n}{dx} + 2nH_n(x) = 0$$

The corresponding generating function $G(t,x) = \sum_{n=0}^{\infty}\frac{1}{n!}H_n(x)t^n$, satisfies the equation:

[CSIR-DEC-2015]

(a) $\frac{\partial^2 G}{\partial x^2} - 2x\frac{\partial G}{\partial x} + 2t\frac{\partial G}{\partial t} = 0$ (b) $\frac{\partial^2 G}{\partial x^2} - 2x\frac{\partial G}{\partial x} - 2t^2\frac{\partial G}{\partial t} = 0$

(c) $\frac{\partial^2 G}{\partial x^2} - 2x\frac{\partial G}{\partial x} + 2\frac{\partial G}{\partial t} = 0$ (d) $\frac{\partial^2 G}{\partial x^2} - 2x\frac{\partial G}{\partial x} + 2\frac{\partial G}{\partial x\partial t} = 0$

16. Let f(x,t) be a solution of the heat equation $\frac{\partial f}{\partial t} = D\frac{\partial^2 f}{\partial x^2}$ in one dimension. The initial condition at t = 0 is $f(x,0) = e^{-x^2}$ for $-\infty < x < \infty$ Then for all $t > 0$, f(x,t) is given by.

[Useful integral: $\int_{-\infty}^{\infty} dx e^{-\alpha x^2} = \sqrt{\frac{\pi}{\alpha}}$] **[CSIR-DEC-2016]**

(a) $\frac{1}{\sqrt{1+Dt}}e^{-\frac{x^2}{1+Dt}}$ (b) $\frac{1}{\sqrt{1+2Dt}}e^{-\frac{x^2}{1+2Dt}}$

(c) $\frac{1}{\sqrt{1+4Dt}}e^{-\frac{x^2}{1+4Dt}}$ (d) $e^{-\frac{x^2}{1+Dt}}$

17. The function y(x) satisfies the differential equation $x\frac{dy}{dx} + 2y = \frac{\cos\pi x}{x}$. If y(1)=1, the value of y(2) is: **[CSIR-JUNE-2017]**

(a) π (b) 1 (c) ½ (d) ¼

18. The Green's function satisfying. **[CSIR-JUNE-2017]**

$$\frac{d^2}{dx^2}g(x,x_0) = \delta(x-x_0)$$

with the boundary conditions $g(-L,x_0) = 0 = g(L,x_0)$, is

(a) $\begin{cases} \frac{1}{2L}(x_0-L)(x+L), -L \le x \le x_0 \\ \frac{1}{2L}(x_0+L)(x-L), \ x_0L \le x \le L \end{cases}$ (b) $\begin{cases} \frac{1}{2L}(x_0+L)(x+L), -L \le x \le x_0 \\ \frac{1}{2L}(x_0-L)(x-L), \ x_0 \le x \le L \end{cases}$

(c) $\begin{cases} \frac{1}{2L}(L-x_0)(x+L), -L \le x \le x_0 \\ \frac{1}{2L}(x_0+L)(L-x), \ \ x_0 \le x \le L \end{cases}$

(d) $\frac{1}{2L}(x-L)(x+L), -L \le x \le L$

19. The number of linearly independent power series solutions, around $x=0$, of the second order linear differential equation $x\frac{d^2y}{dx^2}+\frac{dy}{dx}+xy=0$, is **[CSIR-DEC-2017]**

(a) 0 (this equation does not have a power series solution)

(b) 1 (c) 2 (d) 3

20. The generating function G(t,x) for the Legendre polynomials $P_n(t)$ is. $G(t,x)=\frac{1}{\sqrt{1-2xt+x^2}}=\sum_{n=0}^{\infty}x^nP_n(t)$, for |x|<1 If the function f(x) is defined by the integral equation $\int_0^x f(x')dx'=xG(1,x)$, it can be expressed as. **[CSIR-JUNE-2017]**

(a) $\sum_{n,m=0}^{\infty}x^{n+m}P_n(1)P_m\left(\frac{1}{2}\right)$ (b) $\sum_{n,m=0}^{\infty}x^{n+m}P_n(1)P_m(1)$

(c) $\sum_{n,m=0}^{\infty}x^{n-m}P_n(1)P_m(1)$ (d) $\sum_{n,m=0}^{\infty}x^{n-m}P_n(0)P_m(1)$

21. Consider the following ordinary differential equation $\frac{d^2x}{dt^2}+\frac{1}{x}\left(\frac{dx}{dt}\right)^2-\frac{dx}{dt}=0$ with the boundary conditions $x(t=0)=0$ and $x(t=1)=1$. The value of x(t) at t = 2 is. **[CSIR-JUNE-2018]**

(a) $\sqrt{e-1}$ (b) $\sqrt{e^2+1}$ (c) $\sqrt{e+1}$ (b) $\sqrt{e^2-1}$

22. The Green's function $G(x,x')$ for the equation $\frac{d^2y(x)}{dx^2}+y(x)=f(x)$, with the boundary values $y(0)=y\left(\frac{\pi}{2}\right)=0$, is **[CSIR-JUNE-2018]**

(a) $G(x,x')=\begin{cases} x\left(x'-\frac{\pi}{2}\right), \ \ 0<x<x'<\frac{\pi}{2} \\ \left(x-\frac{\pi}{2}\right)x', \ \ 0<x'<x<\frac{\pi}{2} \end{cases}$

(b) $G(x,x')=\begin{cases} -\cos x' \sin x \ \ \ 0<x<x'<\frac{\pi}{2} \\ -\sin x' \cos x, \ \ 0<x'<x<\frac{\pi}{2} \end{cases}$

(c) $G(x,x')=\begin{cases} \cos x' \sin x, \ \ 0<x<x'<\frac{\pi}{2} \\ \sin x' \cos x, \ \ 0<x'<x<\frac{\pi}{2} \end{cases}$

(d) $G(x,x')=\begin{cases} x\left(\frac{\pi}{2}-x'\right), \ \ 0<x<x'<\frac{\pi}{2} \\ x'\left(\frac{\pi}{2}-x\right), \ \ 0<x'<x<\frac{\pi}{2} \end{cases}$

23. The polynomial $f(x)=1+5x+3x^2$ written as linear combination of the Legendre Polynomials. $\left(P_0(x)=1, P_1(x), P_2(x)=\frac{1}{2}(3x^2-1)\right)$ as $f(x)=\sum_n c_nP_n(x)$. The value of c_0 is. **[CSIR-JUNE-2018]**

(a) 1/4 (b) ½ (c) 2 (d) 4

24. In terms of arbitrary constants A and B , the general solution to the differential equation $x^2\frac{d^2y}{dx^2}+5x\frac{dy}{dx}+3y=0$ is. **[CSIR-DEC-2018]**

(a) $y=\frac{A}{x}+Bx^3$ (b) $y=Ax+\frac{B}{x^3}$

(c) $y = Ax + Bx^3$ (d) $y = \frac{A}{x} + \frac{B}{x^3}$

25. The Green's function $G(x,x')$ for the equation $\frac{d^2y}{dx^2} = f(x)$ with the boundary values $Y(0) = 0$ and $y(1) = 0$, is **[CSIR-DEC-2018]**

(a) $G(x,x') = \begin{cases} \frac{1}{2}x(1-x'), & 0 < x < x' < 1 \\ \frac{1}{2}x'(1-x), & 0 < x' < x < 1 \end{cases}$

(b) $G(x,x') = \begin{cases} x(x'-1), & 0 < x < x' < 1 \\ x'(1-x), & 0 < x' < x < 1 \end{cases}$

(c) $G(x,x') = \begin{cases} -\frac{1}{2}x(1-x'), & 0 < x < x' < 1 \\ \frac{1}{2}x'(1-x), & 0 < x' < x < 1 \end{cases}$

(d) $G(x,x') = \begin{cases} x(x'-1), & 0 < x < x' < 1 \\ x'(x-1), & 0 < x' < x < 1 \end{cases}$

26. The solution of the differential equation $x\frac{dy}{dx} + (1+x)y = e^{-x}$ with the boundary condition y(x=1) = 0, is **[CSIR JUNE-2019]**

(a) $\frac{(x-1)}{x}e^{-x}$ (b) $\frac{(x-1)}{x^2}e^{-x}$ (c) $\frac{(1-x)}{x^2}e^{-x}$ (d) $(x-1)^2e^{-x}$

27. The solution of the differential equation $\left(\frac{dy}{dx}\right)^2 - \frac{d^2y}{dx^2} = e^y$, with the boundary conditions $y(0) = 0$ and $y'(0) = -1$. is **[CSIR NOV-2020]**

(a) $-\ln\left(\frac{x^2}{2} + x + 1\right)$ (b) $-x\ln(e + x)$

(c) $-xe^{-x^2}$ (d) $-x(x+1)e^{-x}$

PROBABILITY

1. An unbiased dice is thrown three times successively. The probability that the numbers of dots on the uppermost surface add up to 16 is. **[CSIR-DEC-2011]**

(a) 1/16 (b) 1/36 (c) 1/108 (d) 1/216

2. A ball is picked at random from one of two boxes that contain 2 black and 3 white and 3 black and 4white balls respectively. What is the probability that it is white? **[CSIR-JUNE-2012]**

(a) 34 / 70 (b) 41/ 70 (c) 36 / 70 (d) 29 / 70

3. A bag contains many balls, each with a number painted on it. There are exactly n balls which have the number n (namely one ball with 1, two balls with 2, and so on until N on them). An experiment consists of choosing a ball at random, noting the number on it and returning it to the bag. If the experiment is repeated a large number of times, the average value the number will tend to. **[CSIR-JUNE-2012]**

S(a) $\frac{2N+1}{3}$ (b) $\frac{N}{2}$ (c) $\frac{N+1}{2}$ (d) $\frac{N(N+1)}{2}$

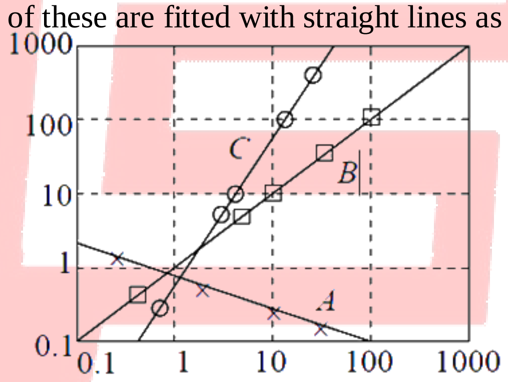
4. In a series of five Cricket matches, one of the captains calls "Heads" every time when the toss is taken. The probability that he will win 3 times and lose 2 times is. **[CSIR-DEC-2012]**
(a) 1/ 8 (b) 5 / 8 (c) 3/16 (d) 5 /16

OTHER QUESTIONS

1. The approximation $\cos\theta \approx 1$ is valid up to 3 decimal places as long as $|\theta|$ is less than:
(take$180^0/\pi \approx 57.29^0$) **[CSIR-JUNE-2013]**
(a) 1.28^0 (b) 1.81^0 (c) 3.28^0 (d) 4.01^0

2. The expression $\left(\frac{\partial^2}{\partial x_1^2}+\frac{\partial^2}{\partial x_2^2}+\frac{\partial^2}{\partial x_3^2}+\frac{\partial^2}{\partial x_4^2}\right)\frac{1}{(x_1^2+x_2^2+x_3^2+x_4^2)}$ is proportional to.
[CSIR- DEC-2013]
(a) $\delta(x_1+x_2+x_3+x_4)$ (b) $\delta(x_1)\delta(x_2)\delta(x_3)\delta(x_4)$
(c) $(x_1^2+x_2^2+x_3^2+x_4^2)^{-3/2}$ (d) $(x_1^2+x_2^2+x_3^2+x_4^2)^{-2}$

3. Three sets of data A, B and C from an experiment, represented by $\times$, $\square$ and O , are plotted on a log-log scale. Each of these are fitted with straight lines as shown in the figure.
[CSIR- DEC-2013]



The functional dependence $y(x)$ for the sets A, B and C are respectively.
(a) $\sqrt{x}, x$ and x^2 (b) $-\frac{x}{2}, x$ and 2x
(c) $\frac{1}{x^2}, x$ and x^2 (d) $\frac{1}{\sqrt{x}}$, x and x^2

4. Two independent random variables m and n , which can take the integer values 0, 1, 2, ...,∞ , follow the Poisson distribution, with distinct mean values μ and v respectively. Then
[CSIR- DEC-2014]
(a) The probability distribution of the random variable l = m + n is a binomial distribution.
(b) The probability distribution of the random variable $r = m - n$ is also a Poisson distribution.
(c) The variance of the random variable $l = m + n$ is equal to $\mu + v$
(d) The mean value of the random variable $r = m - n$ is equal to 0.

5. Let α and β be complex numbers. Which of the following sets of matrices forms a group under matrix multiplication? **[CSIR- DEC-2014]**
(a) $\begin{pmatrix}\alpha & \beta\\ 0 & 0\end{pmatrix}$ (b) $\begin{pmatrix}1 & \alpha\\ \beta & 1\end{pmatrix}$, where $\alpha\beta \neq 1$

(c) $\begin{pmatrix} \alpha & \alpha^* \\ \beta & \beta^* \end{pmatrix}$, where $\alpha\beta^*$ is real

(d) $\begin{pmatrix} \alpha & \beta \\ \beta^* & \alpha^* \end{pmatrix}$, where $|\alpha|^2 + |\beta|^2 = 1$

6. The value of the integral $\int_0^8 \frac{1}{x^2+5} dx$, valuated using Simpson's $\frac{1}{3}$ rule with $h= 2$ is.

[CSIR- DEC-2015]

(a) 0.565 (b) 0.620 (c) 0.698 (d) 0.736

7. Consider a random walker on a square lattice. At each step the walker moves to a nearest neighbour site with equal probability for each of the four sites. The walker starts at the origin and takes 3 steps. The probability that during this walk no site is visited more than one is.

[CSIR- DEC-2015]

(a) 12 / 27 (b) 27 / 64 (c) 3/8 (d) 9 /16

8. Let X and Y be two independent random variables, each of which follow a normal distribution with the same standard deviation σ , but with means $+\mu$ and $-\mu$, respectively. Then the sum $X + Y$ follows a. **[CSIR- JUNE-2016]**

(a) Distribution with two peaks at $\pm\mu$ and mean 0 and standard deviation $\sigma\sqrt{2}$.
(b) Normal distribution with mean 0 and standard deviation 2σ
(c) Distribution with two peaks at $\pm\mu$ and mean 0 and standard deviation 2σ
(d) Normal distribution with mean 0 and standard deviation $\sigma\sqrt{2}$.

9. In finding the roots of the polynomial $f(x) = 3x^3 - 4x - 5$ using the iterative Newton-Raphson method, the initial guess is taken to be $x = 2$. In the next iteration its value is nearest to. **[CSIR-JUNE-2016]**

(a) 1.671 (b) 1.656 (c) 1.559 (d) 1.551

10. Given the values $\sin 45^0 = 0.7071$, $\sin 50^0 = 0.7660$, $\sin 55^0 = 0.8192$ and $\sin 60^0 = 0.8660$, the approximate value of $\sin 52^0$, computed by Newton's forward difference method, is.

[CSIR-DEC-2016]

(a) 0.804 (b) 0.776 (c) 0.788 (d) 0.798

11. The random variable $x(-\infty < x < \infty)$ is distributed according to the normal distribution $P(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{x^2}{2\sigma^2}}$. The probability density of the random variable $y = x^2$ is.

[CSIR-JUNE-2017]

(a) $\frac{1}{\sqrt{2\pi\sigma^2 y}} e^{-\frac{y}{2\sigma^2}}, 0 \le y < \infty$ (b) $\frac{1}{2\sqrt{2\pi\sigma^2 y}} e^{-\frac{y}{2\sigma^2}}, 0 \le y < \infty$

(c) $\frac{1}{\sqrt{2\sigma^2}} e^{-\frac{y}{2\sigma^2}}, 0 \le y < \infty$ (d) $\frac{1}{\sqrt{2\pi\sigma^2 y}} e^{-\frac{y}{2\sigma^2}}, 0 \le y < \infty$

12. Which of the following sets of 3x3 matrices (in which a and b are real numbers) forms a group under matrix multiplication? **[CSIR-JUNE-2017]**

(a) $\left\{\begin{pmatrix} 1 & 0 & a \\ 0 & 1 & 0 \\ b & 0 & 1 \end{pmatrix}; a, b \in \mathbb{R}\right\}$ (b) $\left\{\begin{pmatrix} 1 & a & 0 \\ 0 & 1 & b \\ 0 & 0 & 1 \end{pmatrix}; a, b \in \mathbb{R}\right\}$

(c) $\left\{\begin{pmatrix} 1 & 0 & a \\ 0 & 1 & b \\ 0 & 0 & 1 \end{pmatrix}; a, b \in \mathbb{R}\right\}$ (d) $\left\{\begin{pmatrix} 1 & a & 0 \\ b & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}; a, b \in \mathbb{R}\right\}$

13. A random variable n obeys Poisson statistics. The probability of finding $n = 0$ is 10^{-6}. The expectation value of n is nearest to. **[CSIR-JUNE-2017]**

(a) 14 (b) 10^6 (c) e (d) 10^2

14. Consider an element $U(\varphi)$ of the group SU(2), where φ is any one of the parameters of the group. Under an infinitesimal change $\varphi \to \varphi + \delta\varphi$, it changes as $U(\varphi) \to U(\varphi) + \delta U(\varphi) = (1 + X(\delta\varphi))U(gj)$. To order $\delta\varphi$, the matrix $X(\delta\varphi)$ should always be.

[CSIR- DEC-2016]

(a) positive definite (b) real symmetric
(c) hermitian (d) anti-hermitian

15. The fractional error in estimating the integral $\int_0^1 x dx$ using Simpson's $\frac{1}{3}$ rule, using a step size 0.1 , is nearest to. **[CSIR-JUNE-2018]**

(a) 10^{-4} (b) 0 (c) 10^{-2} (d) 3×10^{-4}

16. The number of distinct ways of placing four indistinguishable balls into five distinguishable boxes is ____________.

17. The value of the integral $\int_{-\pi/2}^{\pi/2} dx \int_{-1}^{+1} dy, \delta(\sin 2x)\delta(x - y)$ is .

[CSIR-JUNE-2018]

(a) 0 (b) ½ (c) $1/\sqrt{2}$ (d) 1

ANSWER KEY

MATRIX ALGEBRA

1.	B	**2.**	A-B, B-A	**3.**	C	**4.**	D	**5.**	A	**6.**	A
7.	B	**8.**	A	**9.**	B	**10.**	B	**11.**	C	**12.**	D
13.	B	**14.**	C	**15.**	D	**16.**	B	**17.**	D	**18.**	D
19.	C	**20.**	A	**21.**	A						

VECTOR ANALYSIS

1.	A	**2.**	B	**3.**	C	**4.**	B	**5.**	All incorrect	**6.**	C
7.	D	**8.**	D	**9.**	A	**10.**					

FOURIER SERIES, FOURIER & LAPLACE TRANSFROMATION

1.	C	**2.**	B	**3.**	C	**4.**	D	**5.**	B	**6.**	A
7.	C	**8.**	B	**9.**	B	**10.**	D	**11.**	A	**12.**	C

COMPLEX ANALYSIS

1.	C	**2.**	B	**3.**	B	**4.**	A	**5.**	B	**6.**	A
7.	D	**8.**	A	**9.**	B	**10.**	C	**11.**	A	**12.**	C
13.	All incorrect	**14.**	A	**15.**	C	**16.**	C	**17.**	D	**18.**	B
19.	C	**20.**	C	**21.**	A	**22.**	C	**23.**	B	**24.**	C
25.	D	**26.**	C	**27.**	C						

DIFFERENTIAL EQUATION

1.	D	**2.**	D	**3.**	D	**4.**	D	**5.**	A	**6.**	A
7.	D	**8.**	A	**9.**	B	**10.**	D	**11.**	C	**12.**	B
13.	D	**14.**	C	**15.**	A	**16.**	D	**17.**	D	**18.**	A
19.	B	**20.**	B	**21.**	C	**22.**	B	**23.**	C	**24.**	D
25.	D	**26.**	A	**27.**	A						

PROBABILITY

1.	B	**2.**	B	**3.**	A	**4.**	D	

OTHER QUESTIONS

1.	B	**2.**	B	**3.**	D	**4.**	C	**5.**	D	**6.**	A
7.	D	**8.**	D	**9.**	B	**10.**	C	**11.**	A	**12.**	C
13.	A	**14.**	D	**15.**	B	**16.**	*	**17.**	B		